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GEOMETRIC MODELLING PLANE CURVES WITH A PARABOLIC CURVATURE UNDER SET ITS DEVIATION FROM THE LINEAR DISTRIBUTION

The work is dedicated to the development of a new approach to the construction of a plane curve line with a parabolic curvature for which curvature is defined by the deviation from the linear distribution of curvature. This problem arises in cases when it is necessary to influence the curvature distribution of the character portion of the flat curve, without changing the value of the curvature in its limit points. Analysis graphics of the parabolic curvature distribution because of its deviation from the linear distribution possible to determine the dependence for calculating the unknown coefficients of the linear parabolic and the curvature distributions. This approach is implemented as a software application on the object-oriented programming language Object Pascal.

Key words: plane curve, curvature, curvature distribution, geometric modelling, parabolic distribution, linear distribution, the deviation.

Research on geometric modeling of plane curves occur in the following areas: architecture and construction, the rational distribution of objects, metal processing, agricultural machinery, fire fighting equipment and technology, aircraft, turbines and compressors, etc.

There are different approaches to geometric modeling curves are flat lines. One of them is based on the notion of integral curve and the model proposed in [4], was further developed in the works of scholars and their students of the Mykolaiv office of the Ukrainian Association on applied geometry which, in particular, deal with issues of geometric modeling of curves and surfaces in relation to the blade turbines and compressors of gas turbine engines. These objects have peculiarities due to the specific working conditions and therefore require the development of specific approaches to the formation of flat sections and on the basis of their surfaces, which limit the course of the working substance, so in this case, important characteristics that are submitted to the stroke, is the continuity of curvature and torsion (spatial lines).

General questions of geometric modeling of the spatial flat and curved contours illuminated in [8]. Flat curved contours, as well as their simulation with the use of graphs of the distribution of curvature is devoted to the publication of [1– 4, 6, 7].

Some approaches to forming of integral curves for a given law of distribution of curvature, given in [1], where a method of computer simulation of planar contours based on the circular spline.

In works of scientists of the Mykolaiv scientific school and their students considered the approach to geometric modeling of curves are flat lines with the use of a given distribution of curvature [2, 3, 6, 7], in this case the boundary conditions are used to build these (or their combination): the coordinates of the start, intermediate and end points of curved contours, the angles of inclination of tangents to the flat curve at these points or some of them, the curvature at the given points.

The distribution of curvature is defined in General terms, for example, in [2] considers linear, in [3] is parabolic, and in [7] is cubic. In one of the reviewed works are not prompted to explore areas of flat curves when you change the character of distribution of curvature.

The aim is to develop a new approach to building a flat curve of parabolic curvature for which curvature is given by the deviation from the linear distribution of curvature. This problem arises in cases when it is necessary to influence the distribution of curvature plot flat curve without changing the values in the curvature of the boundary points.

Consider the plot a flat line curve, shown in fig. 1 [5], where: S – the length of the arc of the plot; ds – the differential of the arc; $\varphi(0)$ – the tangent angle at the starting point; $\varphi(S)$ – the angle of the tangent at the end point of the curve.

This curve corresponds to a certain distribution graph of its curvature $K(s)$, uilt in dependence on the arc length of the contour (fig. 2).

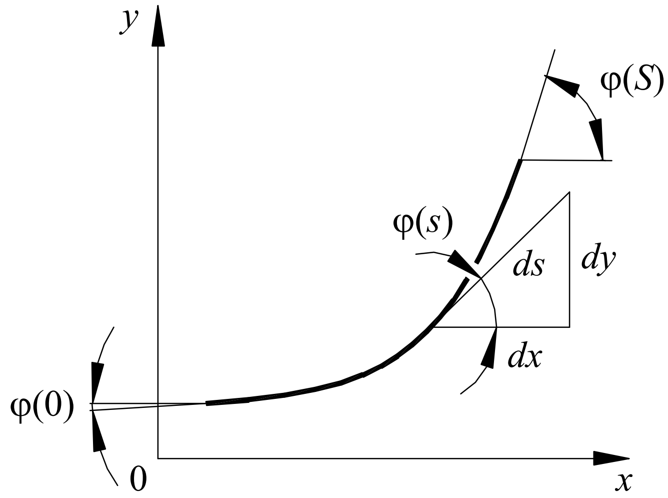

Fig. 1. The plot is flat curve

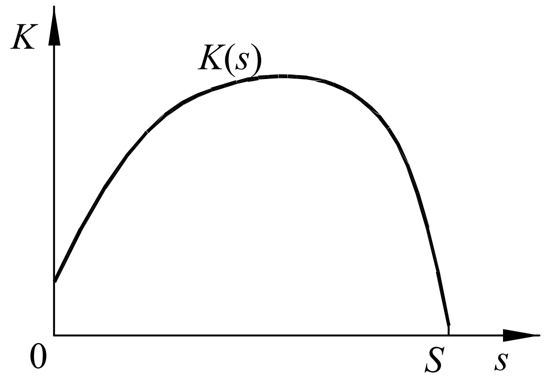

Fig. 2. Schedule of distribution of curvature

If the graph of the distribution of curvature is known, then build a curve that corresponds to it, without any problems. Indeed, the differential of the arc ds at the given values of the tangent angle to the x axis equal to:

$$ds = d\varphi / K(s).$$

Integrating this expression to find the angle of inclination of the tangent to the curve at arbitrary point:

$$\varphi(s) = \varphi(0) + \int_0^s K(s)ds.$$

Consider the case [2] when the curvature of the curve contours along the arc s varies linearly (fig. 3).

Since the curvature of the curve linearly changes from K_1 to K_2, to write the change of curvature in the form of straight line equation in general form:

$$As + BK_1(s) + C = 0,$$

where $A = \dfrac{K_2 - K_1}{S}$, $B = -1$, $C = K_1$, $K_1(s)$ – the dependence of the curvature of the length of the arc.

For the linear law of change of curvature along a curve of a circle the formula for the calculation of the tangent angle will look like this:

$$\varphi_1(s) = \varphi_1(0) + s\left(\frac{As}{2} + C\right).$$

Consider the plot a plane curve [3], which is generated under the condition that the graph is set to a parabolic distribution of the curvature of the curve (fig. 4).

Describe the curve shown in the figure, the parabola of the second degree:

$$K_2(s) = as^2 + bs + c,$$

where $K_2(s)$ – an parabolic dependence of curvature of arc length, $c = K_1$, and the remaining coefficients are based on $aS + b = \dfrac{K_2 - K_1}{S}$.

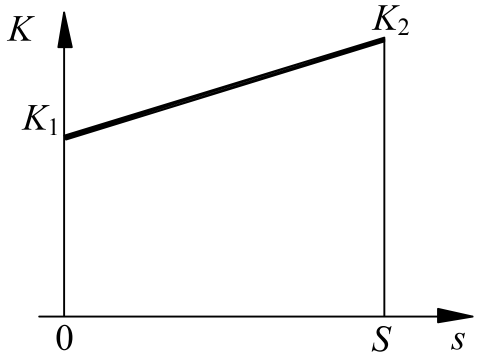

Fig. 3. Graph a linear dependence of curvature of arc length

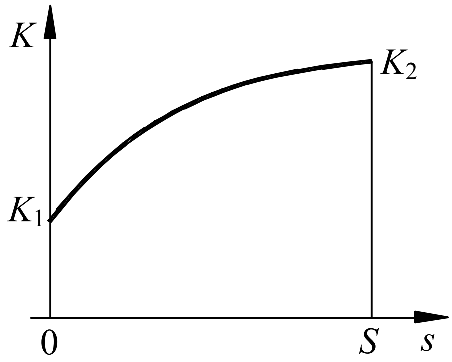

Fig. 4 Graph parabolic distribution of curvature

The angle of the tangent to the curve will be calculated according to the formula:

$$\varphi_2(s) = \varphi_2(0) + s\left(s\left(\frac{as}{3} + \frac{b}{2}\right) + c\right).$$

For determining all coefficients of the parabolic distribution of the curvature will consider its deviations from the linear distribution (fig. 5).

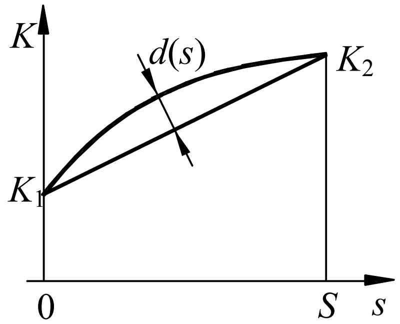

Fig. 5 The deviation of the parabolic distribution of curvature from a linear

The distance between the parabola and straight line, depending on the length of the arc (from analytic geometry) will be determined as follows:

$$d(s) = \frac{\left|As + BK_2(s) + C\right|}{\sqrt{A^2 + B^2}} \qquad (1)$$

The deviation of the parabolic distribution of the curvature of a line will set a maximum distance. For its location, we find the first derivative of an expression (1):

$$d'(s) = \pm\frac{A + B(2as + b)}{\sqrt{A^2 + B^2}}$$

and equate it to zero:

$$A + B(2as + b) = 0.$$

Express the length of the arc at which the derivative is equal to 0 and denote the variable $s_{\max}$:

$$s_{\max} = -\frac{\frac{A}{B} + b}{2a}. \qquad (2)$$

Substitute the expression (2) to (1) and after transformation we find the maximum deviation of the parabolic distribution of curvature from a linear:

$$d_{\max} = \frac{aS^2}{4}.$$

Hence the unknown factor *a* of a parabolic distribution of curvature is equal to:

$$a = \frac{4d_{\max}}{S^2}.$$

Then

$$b = \frac{K_2 - K_1 - 4d_{\max}}{S}.$$

Thus, the obtained expressions to obtain the unknown coefficients of the parabolic distribution of the curvature, taking into account its deviation from the linear distribution.

In the fig. 6 shows the curvature distribution for different values of maximum deviations parabolic curvature distribution of linear distribution. In this case on fig. 6, *a* shows the distribution of curvature when $d_{\max} = -0{,}3; -0{,}4; -0{,}5$, and fig. 6, б – when $d_{\max} = 0{,}3; 0{,}4; 0{,}5$.

In the pictures the order in which the curvature corresponds to the order given in the descriptions to the pictures.

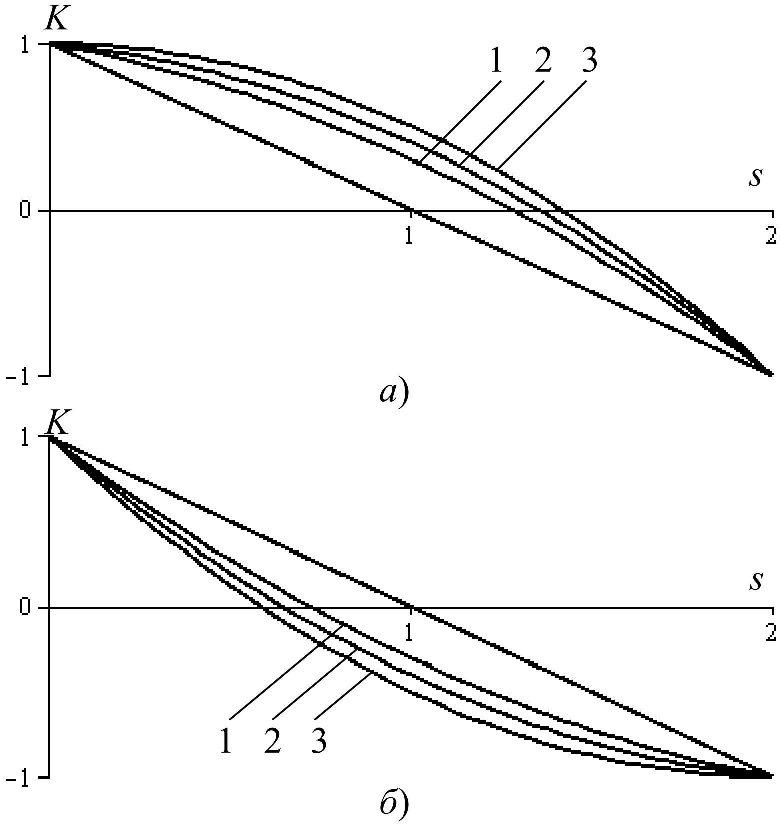

Fig. 6 Distribution of curvature depending on the maximum deflection of a parabolic distribution of curvature from a linear

Find the equation of the curve, which is formed on the basis of a given distribution of curvature. With fig. 1 it follows that

$$dx = ds\cos\varphi(s);\ dy = ds\sin\varphi(s).$$

Integrating these expressions we obtain the parametric equations of the curve, which made for setting arc length of the arc:

$$x(s) = x(0) + \int_0^s \cos\varphi(s)\,ds;$$

$$y(s) = y(0) + \int_0^s \sin\varphi(s)\,ds,$$

where $x(0)$, $y(0)$ – the coordinates of the start point of the curve.

These equations are the equations of the spiral, and the integrals are evaluated by numerical methods, e.g. Simpson method.

The parametric equation of the curved area of the rim, obtained with a linear law of change of curvature will look like this:

$$x_1(s) = x_1(0) + \int_0^s \cos\left(\varphi_1(0) + s\left(\frac{As}{2} + C\right)\right)ds;$$

$$y_1(s) = y_1(0) + \int_0^s \sin\left(\varphi_1(0) + s\left(\frac{As}{2} + C\right)\right)ds,$$

an using a parabolic distribution of curvature – this:

$$x_2(s)=x_2(0)+\int_0^s \cos\left(\varphi_2(0)+s\left(s\left(\frac{as}{3}+\frac{b}{2}\right)+c\right)\right)ds;$$

$$y_2(s)=y_2(0)+\int_0^s \sin\left(\varphi_2(0)+s\left(s\left(\frac{as}{3}+\frac{b}{2}\right)+c\right)\right)ds.$$

For the analytical representation of the contour applies a parabolic distribution of the curvature with a predetermined deviation from a linear distribution $d_{\max}$, unknown primary K_1 and end K_2 meaning curvature and arc length curve S.

Write parametric equations of the curve and substitute in them the coordinates of the start and end points, arc length, and expressions for determining the coefficients of the parabolic distribution of curvature. After transformation we have the following system that consists of two equations:

$$x_1=x_0+\int_0^S \cos\left(\varphi_0+s\left\{\frac{s}{S}\left[2d_{\max}\left(\frac{2s}{3S}-1\right)+\frac{K_2-K_1}{2}\right]+K_1\right\}\right)ds;$$

$$y_1=y_0+\int_0^S \sin\left(\varphi_0+s\left\{\frac{s}{S}\left[2d_{\max}\left(\frac{2s}{3S}-1\right)+\frac{K_2-K_1}{2}\right]+K_1\right\}\right)ds.$$

In this system of equations three unknowns. For its numerical solution we write the equation of determining the angle of inclination of the tangent to the curve at the end point and substitute into it the expression to determine the coefficients of the parabolic distribution. After a number of transformations we obtain a formula for the determination of the curvature at the endpoint of the curve:

$$K_2=2\left(\frac{\Delta\varphi}{S}+\frac{2}{3}d_{\max}\right)-K_1.$$

This formula will substitute the system of parametric equations of the curve obtained before and after the transformation will have:

$$x_1=x_0+\int_0^S \cos\left(\varphi_0+\frac{s^2}{S}\left[\frac{\Delta\varphi}{S}-K_1-\frac{4}{3}d_{\max}\left(1-\frac{s}{S}\right)\right]+K_1 s\right)ds;$$

$$y_1=y_0+\int_0^S \sin\left(\varphi_0+\frac{s^2}{S}\left[\frac{\Delta\varphi}{S}-K_1-\frac{4}{3}d_{\max}\left(1-\frac{s}{S}\right)\right]+K_1 s\right)ds.$$

The resulting system of equations can only be solved numerically, e.g. with Newton's method. But you need to determine the derived equations for the unknown parameters S i K_1:

$$\frac{\partial f_1}{\partial K_1}=-\frac{1}{S}\int_0^S s(S-s)\sin\Phi(s)ds;$$

$$\frac{\partial f_2}{\partial K_1}=\frac{1}{S}\int_0^S s(S-s)\cos\Phi(s)ds;$$

$$\frac{\partial f_1}{\partial S}=\frac{1}{S}\int_0^S\left[\cos\Phi(s)-\left(S\frac{\partial\Phi}{\partial S}+s\frac{\partial\Phi}{\partial s}\right)\sin\Phi(s)\right]ds;$$

$$\frac{\partial f_2}{\partial S}=\frac{1}{S}\int_0^S\left[\sin\Phi(s)+\left(S\frac{\partial\Phi}{\partial S}+s\frac{\partial\Phi}{\partial s}\right)\cos\Phi(s)\right]ds,$$

where

$$\Phi(s)=\varphi_0+\frac{s^2}{S}\left[\frac{\Delta\varphi}{S}-K_1-\frac{4}{3}d_{\max}\left(1-\frac{s}{S}\right)\right]+K_1 s;$$

$$\frac{\partial \Phi}{\partial S} = \frac{s^2}{S^2}\left[-\frac{2\Delta\varphi}{S} + K_1 + \frac{4}{3} d_{\max}\left(1 - \frac{2s}{S}\right)\right]$$

$$\frac{\partial \Phi}{\partial s} = \frac{2s}{S}\left[\frac{\Delta\varphi}{S} - K_1 - 2d_{\max}\left(\frac{2}{3} + \frac{s}{S}\right)\right] + K_1 .$$

For the practical implementation of the proposed method of constructing a curvilinear contour of a given curvature developed by the software in the language of object-oriented programming Object Pascal in the visual design Delphi.

In fig. 7 shows the curves that were modeled with a variable value of the maximum deviation of the parabolic distribution of curvature from a linear distribution.

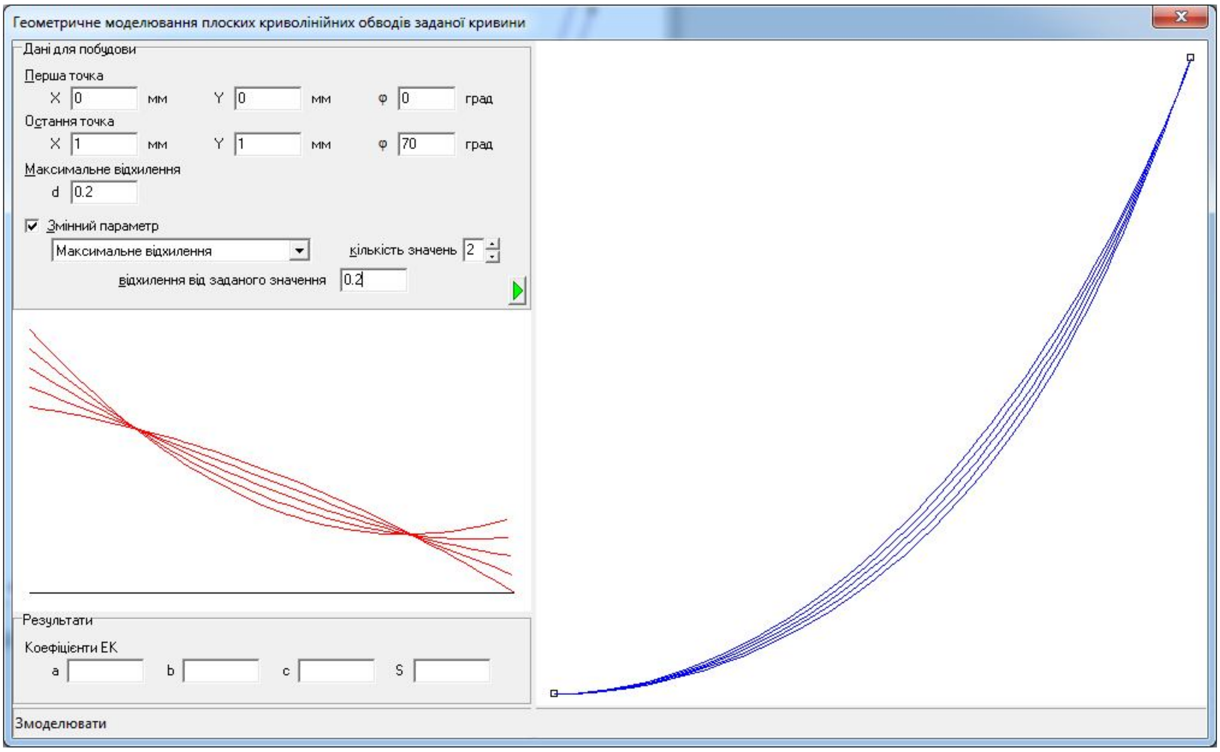

Fig. 7 The curves obtained for different values of the maximum deflection of a parabolic distribution of curvature from a linear distribution

The result of geometric modeling a plane curve with a parabolic criminal given it deviated from the linear distribution of the resulting integrated system of parametric equations describing this curve and provides its passage through the two points specified in their angles of inclination of tangents. Further research will be directed toward finding the optimal values of the maximum deflection of a parabolic distribution of curvature from a linear deviation.

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ГЕОМЕТРИЧНЕ МОДЕЛЮВАННЯ ПЛОСКОЇ КРИВОЇ ІЗ ПАРАБОЛІЧНОЮ КРИВИНОЮ ПРИ ЗАДАНОМУ ЇЇ ВІДХИЛЕНІ ВІД ЛІНІЙНОГО РОЗПОДІЛУ

Робота присвячена розробці нового підходу до побудови плоскої кривої лінії із параболічною кривиною, для якої задається відхилення кривини від лінійного розподілу кривини. Така задача виникає у випадках коли потрібно впливати на характер розподілу кривини ділянки плоскої кривої лінії, не змінюючи при цьому значення кривини в її граничних точках. Дослідження графіку параболічного розподілу кривини з урахуванням його відхилення від лінійного розподілу дозволило визначити залежності для обчислення невідомих коефіцієнтів параболічного та лінійного розподілів кривини. Запропонований підхід реалізовано у вигляді програмного додатку об'єктно-орієнтованою мовою програмування Object Pascal.

Ключові слова: плоска крива, кривина, розподіл кривини, геометричне моделювання, параболічний розподіл, лінійний розподіл, відхилення.

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г. Николаев

ГЕОМЕТРИЧЕСКОЕ МОДЕЛИРОВАНИЕ ПЛОСКОЙ КРИВОЙ С ПАРАБОЛИЧЕСКОЙ КРИВИЗНОЙ ПРИ ЗАДАННОМ ЕЕ ОТКЛОНЕНИИ ОТ ЛИНЕЙНОГО РАСПРЕДЕЛЕНИЯ

Работа посвящена разработке нового подхода к построению плоской кривой линии с параболической кривизной, для которой задается отклонение кривизны от линейного распределения кривизны. Такая задача возникает в случаях, когда нужно воздействовать на характер распределения кривизны участка плоской кривой линии, не меняя при этом значение кривизны в ее предельных точках. Исследование графика параболического распределения кривизны с учетом его отклонения от линейного распределения позволило определить зависимости для вычисления неизвестных коэффициентов параболического и линейного распределений кривизны. Предложенный подход реализован в виде программного приложения на объектно-ориентированном языке программирования Object Pascal.

Ключевые слова: плоская кривая, кривизна, распределение кривизны, геометрическое моделирование, параболическое распределение, линейное распределение, отклонение.

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